GCPC 2022
Presentation of solutions

Jury and Testers

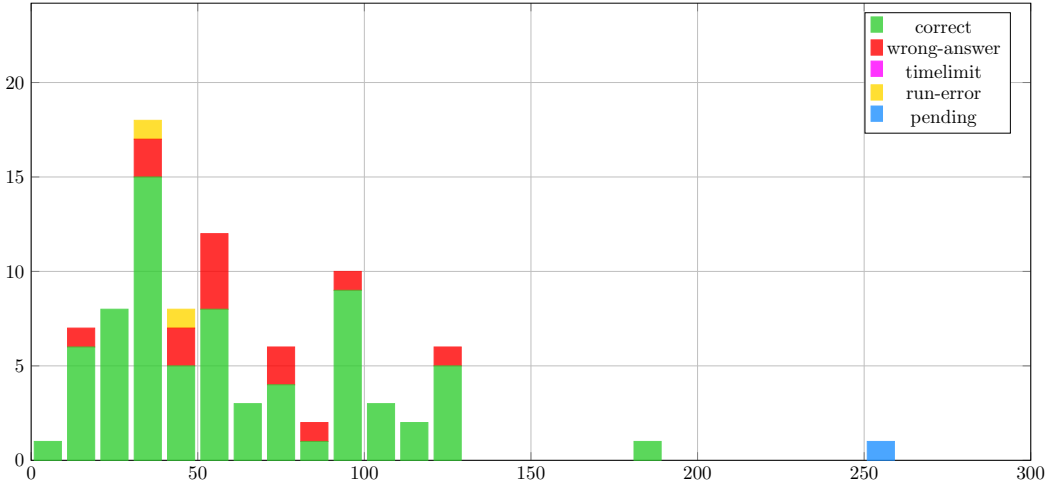
Thanks to the jury:

- Lucas Alber (KIT)
- Julian Baldus (Saarbrücken)
- Paul Jungeblut (KIT)
- Felicia Lucke (CPUlm)
- Nathan Maier (CPUlm)
- Jannik Olbrich (CPUlm)
- Michael Ruderer (CPUlm)
- Marcel Wienöbst (Lübeck)
- Paul Wild (FAU)
- Wendy Yi (KIT)
- Michael Zündorf (KIT)

Thanks to our test readers:

- Andreas Grigorjew (Helsinki)
- Hans Spath

K – K.O. Kids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Problem

k players walk over a bridge of length n, where each step consists of one real and one fake plate. The i-th player knows the real plate for all steps entered by players $1, \ldots, i-1$. All players switch sides at each step, the first player begins with the left plate. How many players make it over the bridge for a given layout s?

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Solution

- Game can be simulated naively in $O(n \cdot k)$ and also in $O(n + k)$ by starting at the last known step. This is fast enough.

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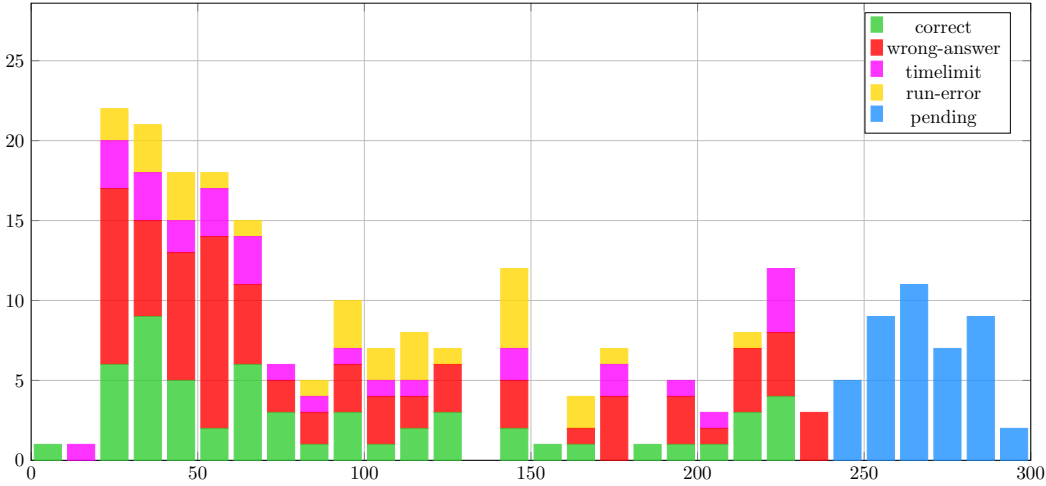
Solution

- Game can be simulated naively in $O(n \cdot k)$ and also in $O(n + k)$ by starting at the last known step. This is fast enough.
- An $O(n)$ solution is possible as well:
  - Add an imaginary zeroth step $s[0] = \mathtt{R}$ to handle the first player starting on the left.
  - Let d be the number of steps with $s[i] = s[i-1]$ for $i = 1, \ldots, n$.
  - The solution is $\max(k - d, 0)$.

E – Enjoyable Entree



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A canteen has a soup on offer whose recipe changes every day:

- On each of the first two days, the soup has some given type.
- On each subsequent day, the soup is an equal mix of the two previous days' soups.

Find the percentages of the first two days' soups that can be found in the nth soup.

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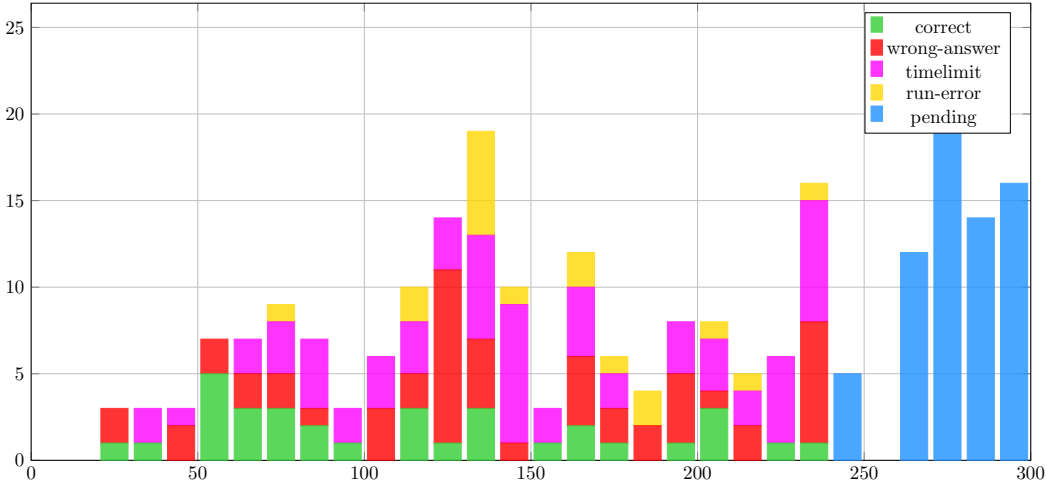
Solution

- We need to solve the following recurrence:

$$v_1 = (1, 0) \qquad v_2 = (0, 1) \qquad v_n = \tfrac{1}{2}(v_{n-1} + v_{n-2}) \qquad (n \geq 3)$$

- The number n may be very large, so that simple iteration will be too slow.
- However, the sequence quickly converges to its limit, $(\frac{1}{3}, \frac{2}{3})$.
- If we stop after 100 steps, the answer will have the required precision of 10^{-6}.

C – Chaotic Construction



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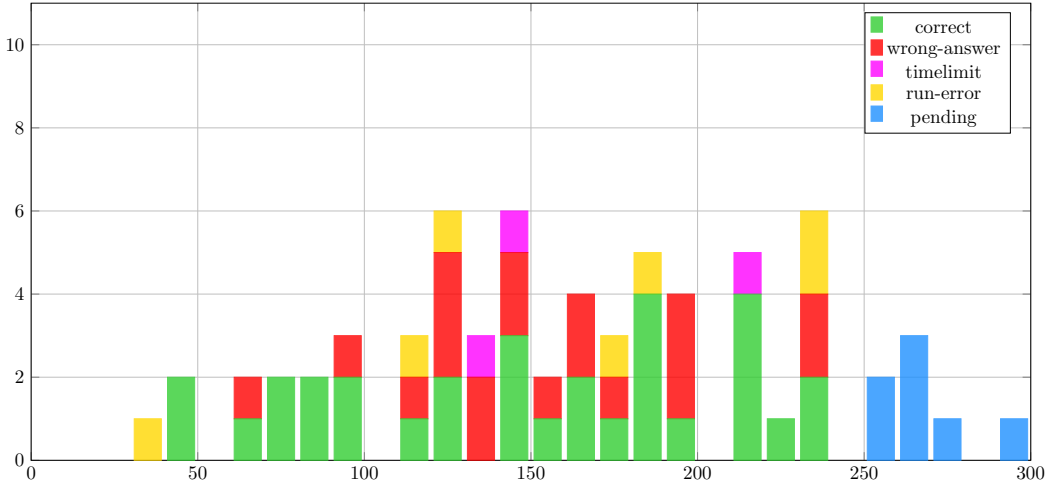
Solution

- Store blocked positions in a balanced binary search tree / segment tree.
- Use the tree to determine in $O(\log n)$ whether there is a blocked road ...
  - ... between a and b
  - ... at a position $\geq b$ or $\leq a$

If both conditions are met, the result is `impossible`, otherwise it is `possible`.

I – Improving IT



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Given a list of CPU prices over n months and their resale values for the following m months, find the lowest cost to operate the system over n months.

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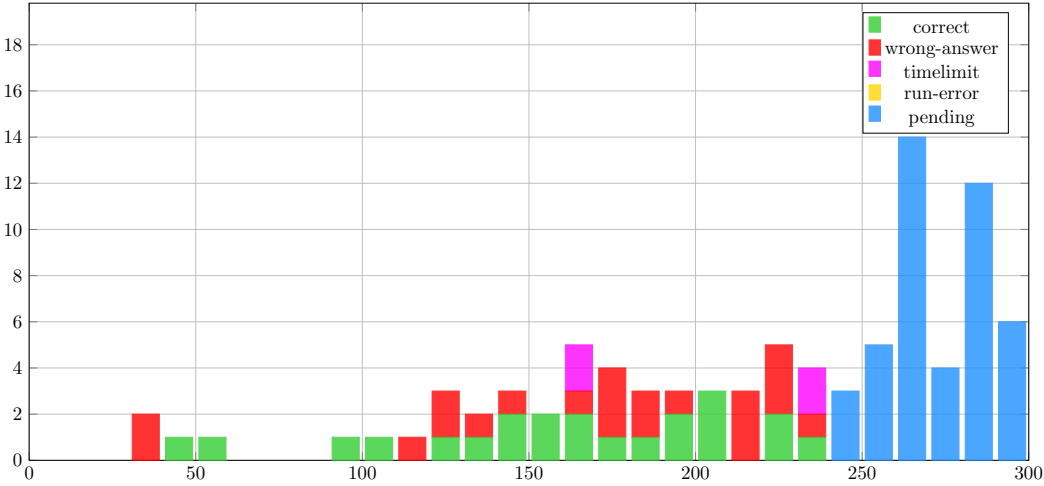
- The problem can be transformed into the shortest path problem:
- Build a graph with nodes $1, \ldots, n+1$ and edges $(i, i+j)$ with weights $d_{i,j}$, where $d_{i,j}$ is the cost for buying a CPU in month i and keeping it for j months.
- Compute the shortest path from node 1 to $n+1$ by visiting the nodes in order $1, \ldots, n+1$.

D – Diabolic Doofenshmirtz



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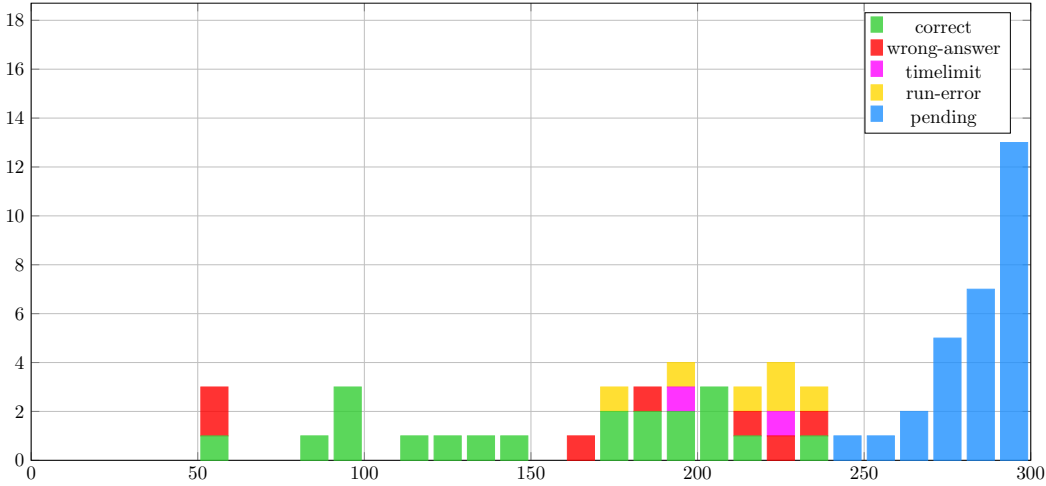
$\Rightarrow$ Try $t = 2^i - 1$ until $t \bmod x \neq t$. Then answer with $t - (t \bmod x)$.

H – Hardcore Hangman



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Hangman with the possibility to query multiple letters at once. Queries are answered with the set of all indices containing one of the queried letters. Find the hidden word with at most 7 queries.

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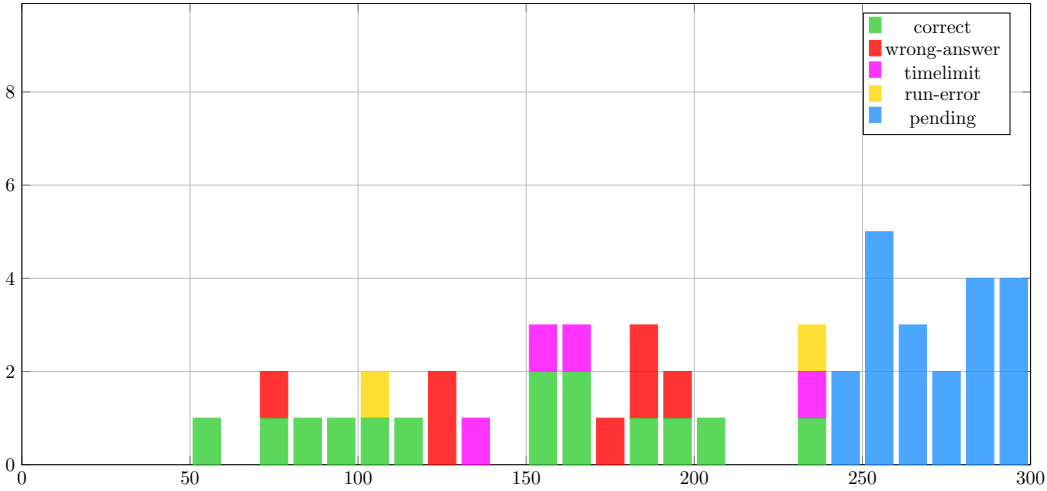
- To reconstruct the word, find, for each letter ℓ, the indices which are in the answers of the queries with ℓ included and not in the other ones.
- Runtime is $O(n)$ for a hidden word of length n and the strategy uses exactly 7 queries. (A solution with 6 queries exists as well.)

L – Lots of Land



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Given integers h, w, n ($1 \leq h, w \leq 100$, $1 \leq n \leq 26$), find a way to split an $h \times w$ rectangle into n rectangular parts of equal area and with integer side lengths.

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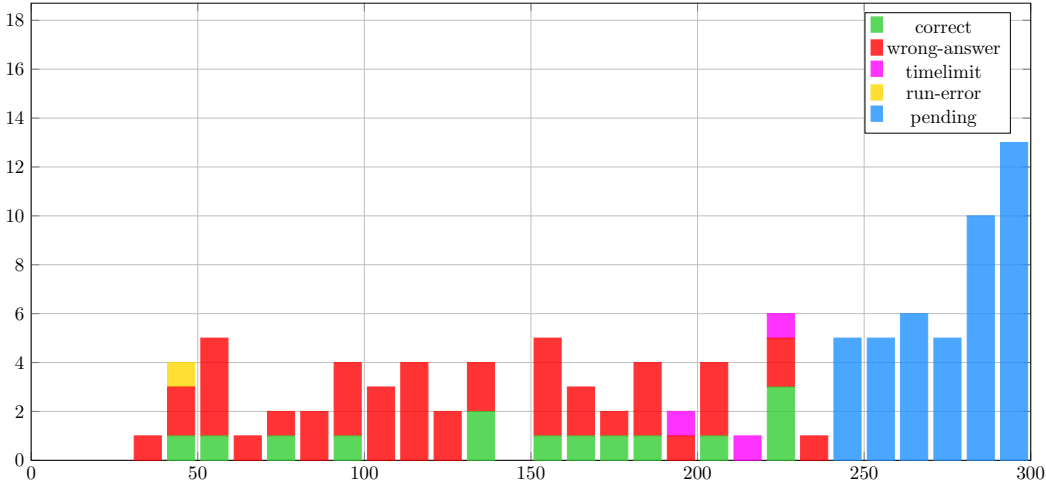
Solution

- If $h \cdot w$ is not divisible by n, no solution exists, so output `impossible`.
- Otherwise, it's always possible to split the large rectangle into a *grid* of rectangles.
- Can divide into $a \times b$ rectangles $\iff$ a divides h, b divides w and $n \cdot a \cdot b = h \cdot w$.
- Once a and b are found, many possible ways to construct the answer, e.g.:

```
answer[i][j] = 'A' + (i/a)*(w/b) + (j/b)
```

A – Alternative Architecture



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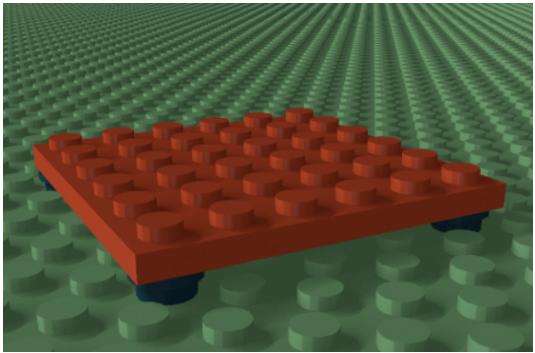
How many ways are there to place a Lego plate of dimensions $a \times b$ on an infinite base plate, if smaller 1×1 plates placed in the corners are used as buffer?



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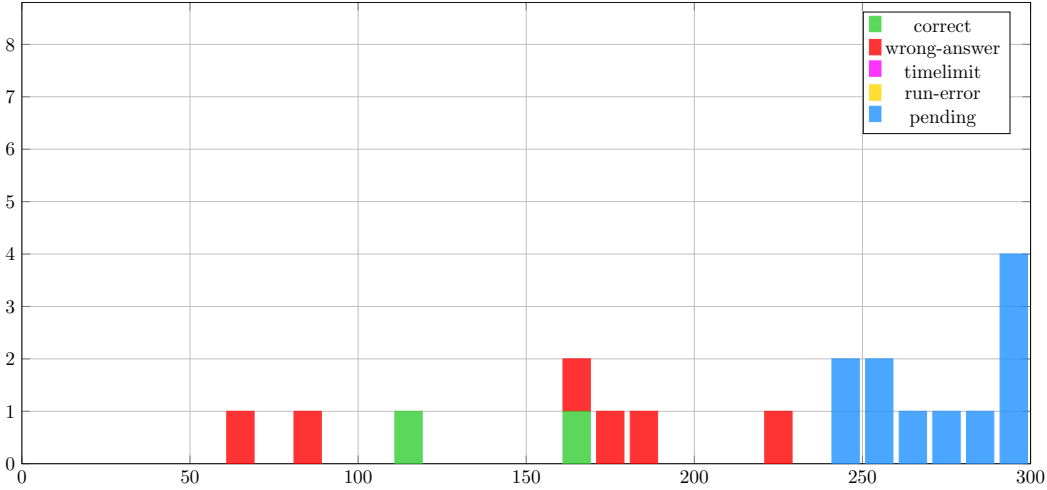
Equivalent Problem

Find the number of ways to draw an $(a-1) \times (b-1)$ rectangle in the plane so that all of its corners have integer coordinates.

Solution

- Place one of the corners at $(0, 0)$.
- The next corner must be at some point (x, y) with $x^2 + y^2 = (a-1)^2$.
- Find all candidate points, e.g. by first looping over all x and then solving for y.
- Compute the coordinates of the two other corners and check if they are integral.
- Make sure to avoid double counting if the rectangle is a square ($a = b$).

J – Jesting Jabberwocky



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Solution

- Observe that since we can reinsert a card at any position, we only have to count how many cards need to be removed.
- For a fixed order of suits, we can calculate the minimum number of cards to be removed using dynamic programming.

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Solution

Dynamic programming for a fixed order of suits:

- Let `dp[i][s]` be the minimum number of cards to remove such that the remaining of the first i cards are ordered correctly and the last card is of suit s.

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- Let `dp[i][s]` be the minimum number of cards to remove such that the remaining of the first i cards are ordered correctly and the last card is of suit s.
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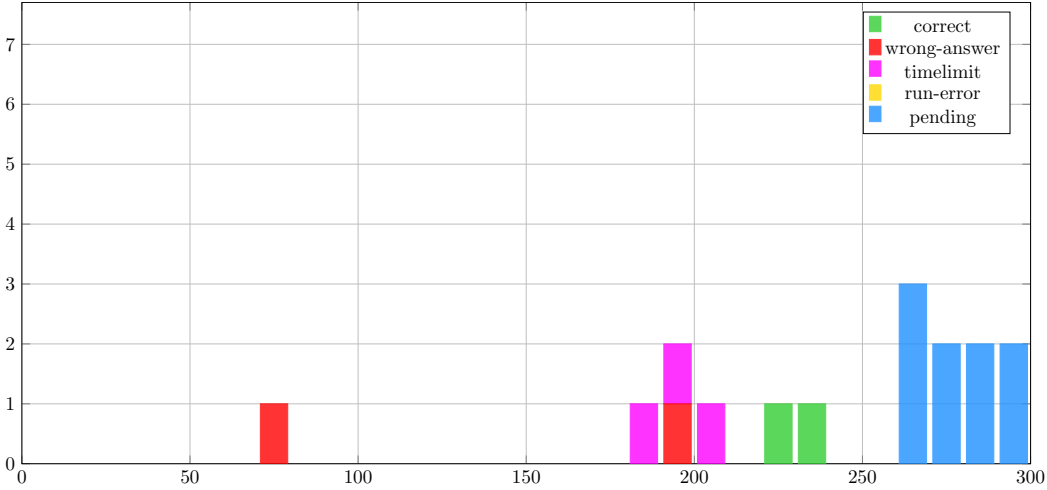
This problem can also be solved in $\mathcal{O}(k \cdot 2^k \cdot n)$.

B – Breeding Bugs



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Now observe the following properties:

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$\Rightarrow$ The given graph is *bipartite*.

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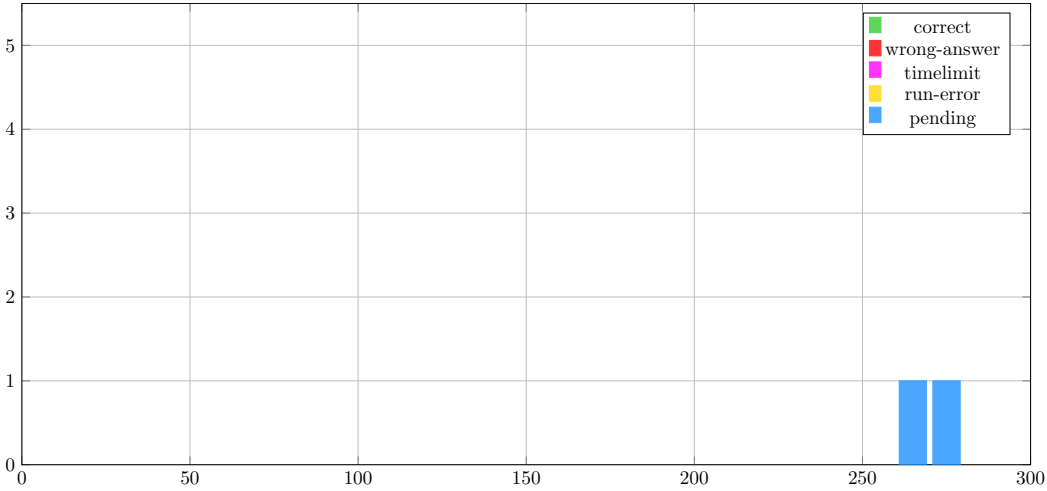
- A maximum matching can be found in many ways:
  - Any flow algorithm
  - Kuhn's algorithm
  - Hopcroft Karp's algorithm

M – Mirror Madness



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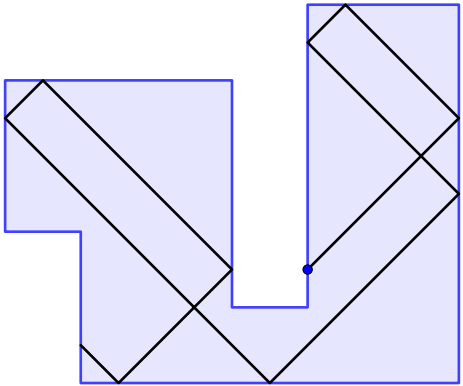
Problem

Simulate the path of a laser beam inside an axis-aligned polygon.



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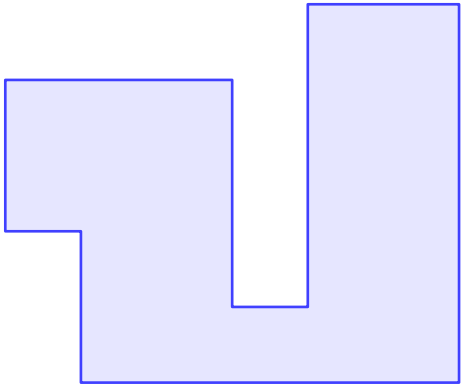

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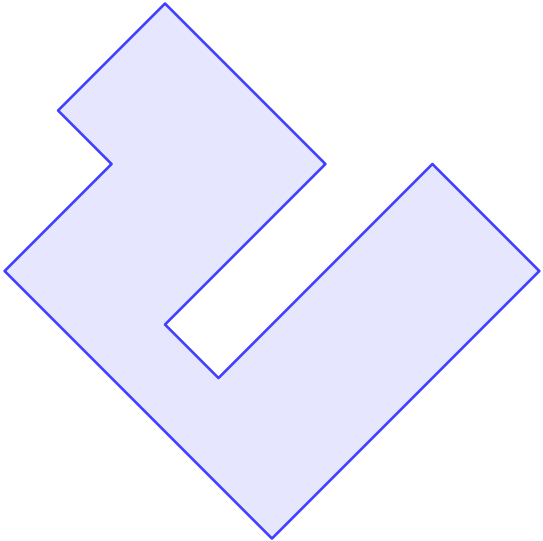

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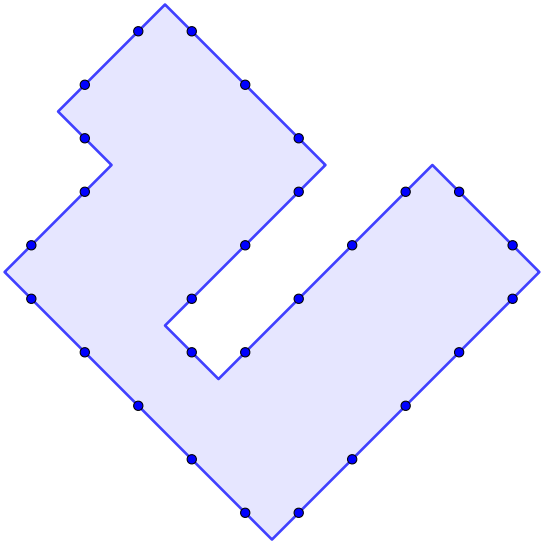

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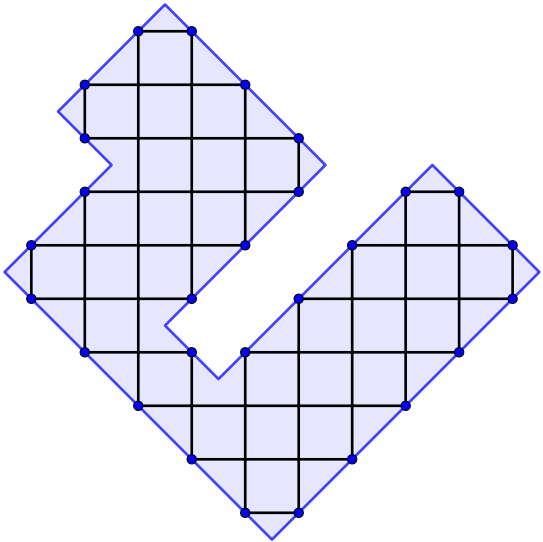
- Connect them up by forming pairs within each row and column.



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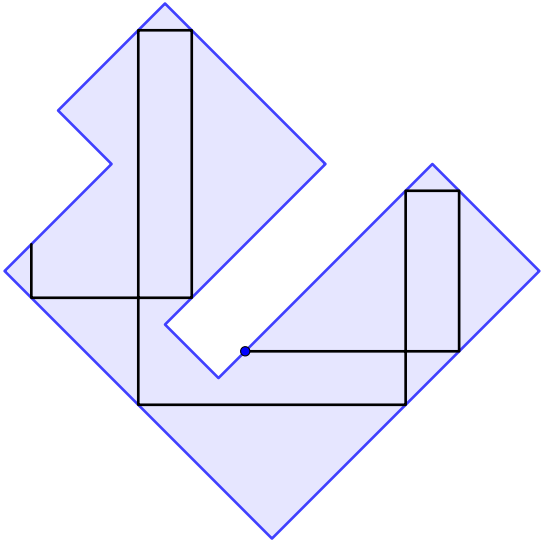
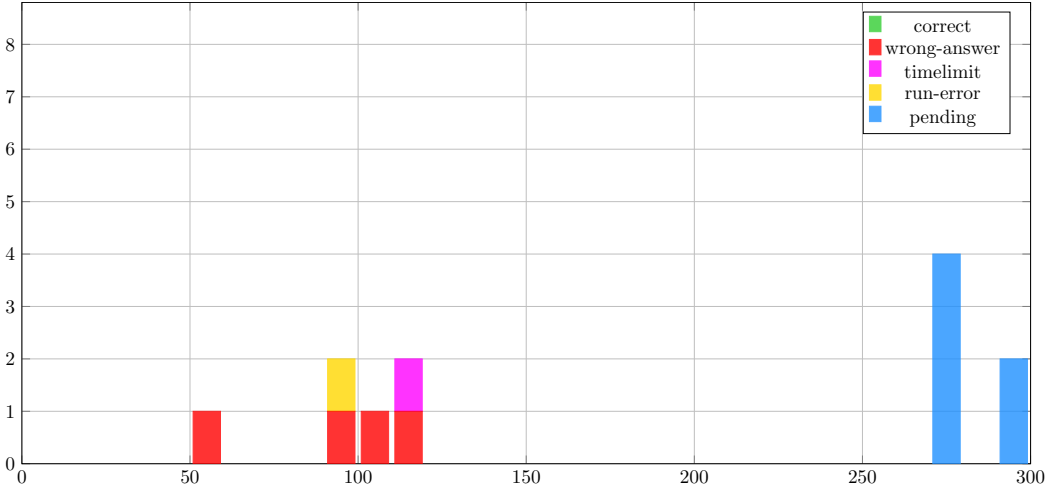
- Construct the answer step by step by following the pointers between pairs.



G – Guessing Game



G – Guessing Game

Problem

- There is a pool of yes-or-no events (the examinations).
- We are given a list of guesses. There are n guesses each containing predictions of seven distinct events. Alan guesses the outcome of two distinct events.
- Is there an outcome of the events where Alan's guess contains more *correct* predictions than any other guess?

G – Guessing Game

Problem

Is there an outcome of the events where Alan's guess contains more *correct* predictions than any other guess?

Solution

- If there is such an outcome, there is one where all of Alan's predictions come true
- Then it suffices to ensure that no other guess contains more than one correct prediction

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- We encode these two conditions into a 2SAT formula

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Is there an outcome of the events where Alan's guess contains more *correct* predictions than any other guess?

Solution

- Each event has two possible results.
  $\Rightarrow$ Boolean variable X_e for all events e
- Prediction P for event e can be expressed as either X_e or $\neg X_e$
- "Alan predictions [P_1 and P_2] must both be correct" $\Rightarrow P_1 \wedge P_2$
- "A following submission [consisting of $P'_1, \ldots, P'_7$] must not contain two correct predictions"

$$\Rightarrow \bigwedge_{1 \leq i < j \leq 7} (\neg P'_i \vee \neg P'_j)$$

G – Guessing Game

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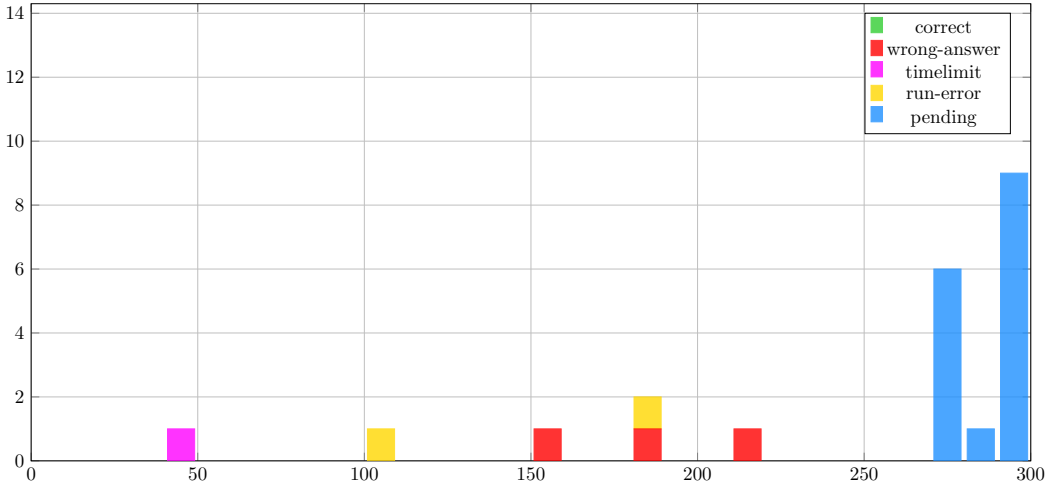
Is there an outcome of the events where Alan's guess contains more *correct* predictions than any other guess?

Solution

$$\Rightarrow \quad P_1 \wedge P_2 \wedge \bigwedge_{k=1}^{n} \bigwedge_{i=1}^{7} \bigwedge_{j=i+1}^{7} \left(\neg P_i^{(k)} \vee \neg P_j^{(k)} \right)$$

- Alan can win if and only if this formula is satisfiable
- Use your favourite 2SAT algorithm to check this
- Running time: $\mathcal{O}(n)$

F – Formula Flatland



F – Formula Flatland

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Given a planar graph with minimum degree at least 3, find the length of the smallest circle. This value is also called the *girth* of a graph.

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- We start with an edge, take the head vertex of the edge, find the next edge in cyclic order and continue.

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- The smallest face we see is the answer.
- Runtime: $\mathcal{O}(n \log(n))$ for sorting and $\mathcal{O}(n)$ to enumerate all faces.

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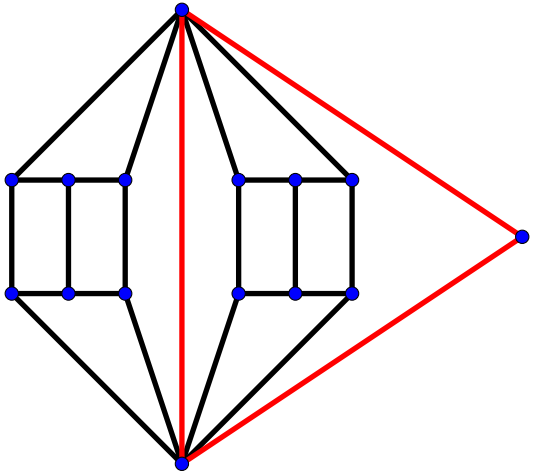

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Given a planar graph with minimum degree at least 3, find the smallest circle. This property is also called the *girth* of a graph.

Real Solution

F – Formula Flatland

Problem

Given a planar graph with minimum degree at least 3, find the smallest circle. This property is also called the *girth* of a graph.

Real Solution

- Orient the edges such that each vertex has outdegree ≤ 5 and the graph is a DAG.
- Can be done by recursively removing a vertex with degree ≤ 5.

F – Formula Flatland

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Real Solution

- Orient the edges such that each vertex has outdegree ≤ 5 and the graph is a DAG.
- Can be done by recursively removing a vertex with degree ≤ 5.
- Observe that the girth is either 3, 4 or 5.

$\Rightarrow$ We only need to check if a cycle of length 3 or 4 exists.

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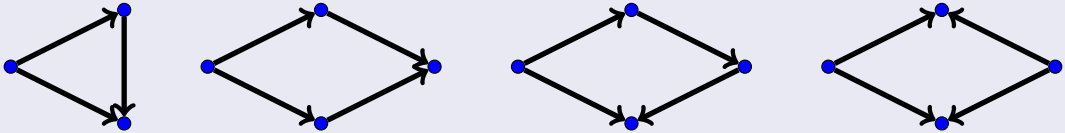

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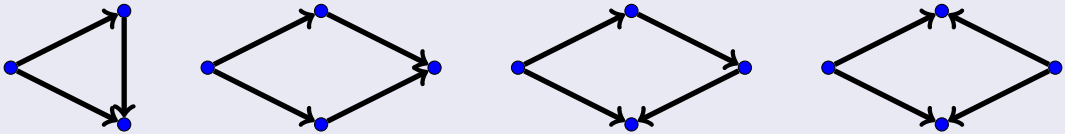

- The first three cases can be solved with a limited BFS from each vertex.
- We go until depth 3, so the runtime is $\mathcal{O}(5^3 n)$.

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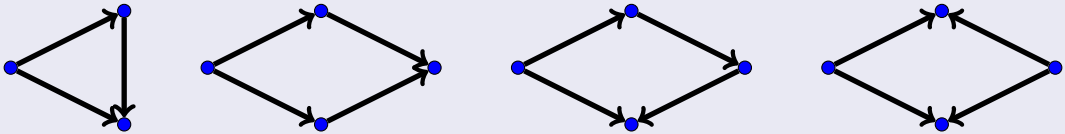

- The fourth case can be solved by enumerating all pairs of outgoing edges and checking for duplicates.
- We have limited degree, so the runtime is $\mathcal{O}(5^2 n \cdot \log(n))$.